4.1

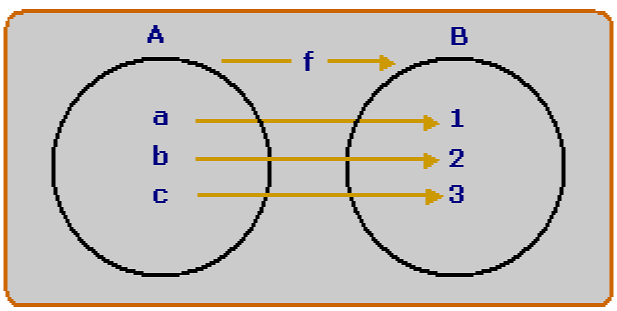
Functions and Their Properties

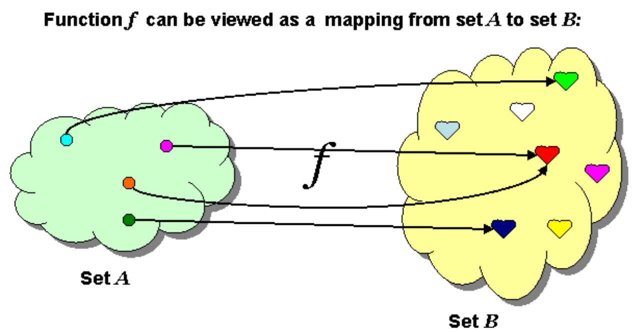
<u>Function, Domain, and Range:</u> A function from set *D* to a set *R* is a rule that assigns to every element in *D* a unique element in *R*. The set *D* of all input values is the **domain** of the function, and the set *R* of all output values is the **range** of the function.

Function notation $y = f(x)$ "*y* equals f of *x*" or "the value of *f* at *x*".

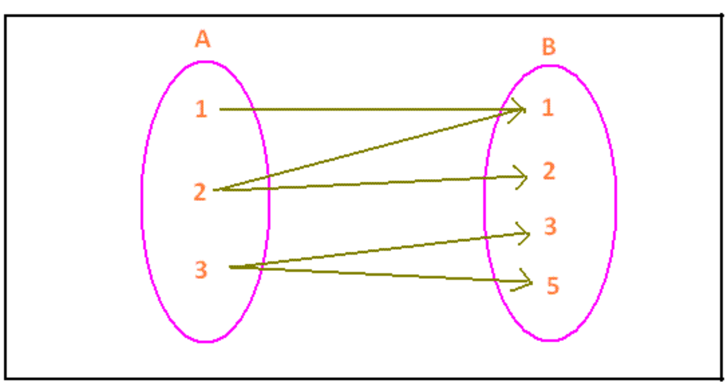
Function as a mapping or a machine.

Function

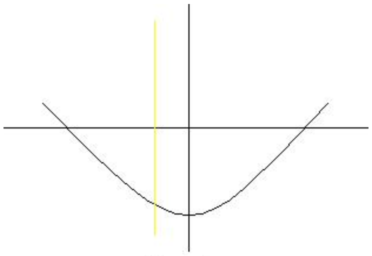




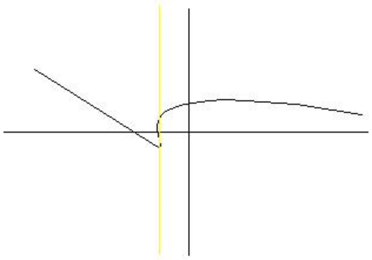
Not a function



<u>Vertical Line Test:</u> A graph (set of points (*x, y*)) in the *xy*-plane defines *y* as a function of *x* if and only in no vertical line intersects the graph in more than one point.

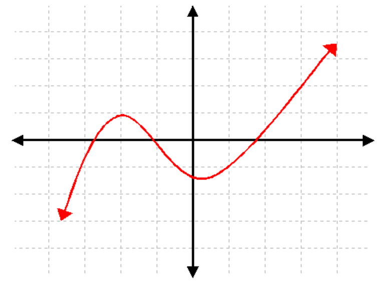


Function



Not a Function=

Function



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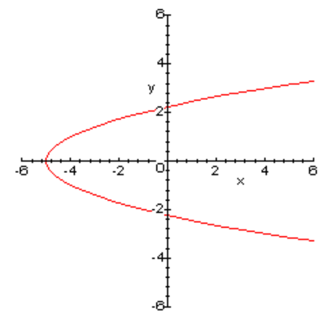


Agreement implied domain – same as domain of the algebraic expression.

Relevant domain – domain that fits the situation

Find domain of each function.

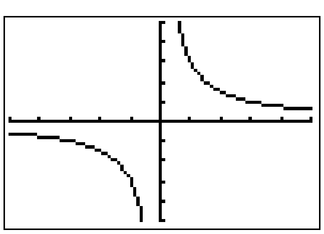
a) $f(x)=\sqrt{x+3}$; $\quad x\geq -3$

b) $g(x)=\frac{\sqrt{x}}{x-5}$; $\quad x\geq 0,\ x\neq 5$

Finding Range – look for all *y*-coordinates that correspond to points on the graph.

Find the range of the function $f(x)=\dfrac{3}{x}$.

The graph of $f(x)=\dfrac{3}{x}$ is shown below.



In the graph it appears that the range consists of all real number except 0. This can be confirmed algebraically by solving:

$$\frac{3}{x}=0$$
$$x\bullet\frac{3}{x}=0\bullet x$$
$$3=0$$

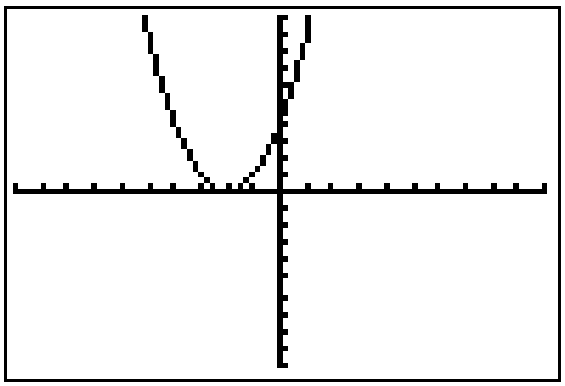
A function is:

Increasing on an interval if, for any two points in the interval a positive change in *x* results in a positive change in *f(x)*

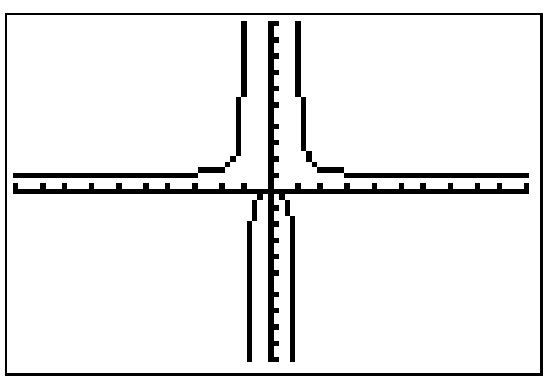
Decreasing on an interval if, for any two points in the interval a positive change in *x* results in a negative change in *f(x)*

Constant on an interval if for any two points in the interval a positive change in *x* results in a zero change in *f(x)*

Ex. $f(x) = (x + 2)^2$



Ex. $f(x) = \dfrac{x^2}{x^2 - 1}$



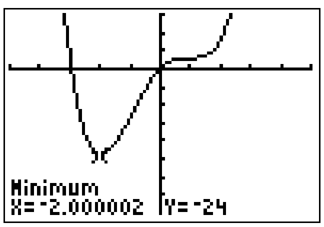
A local maximum of a function *f* is a value $f\,(c)$ that is greater than or equal to all range values of f on some open interval containing c. If $f(c)$ is greater than or equal to all range values of f then $f(c)$ is the max (or absolute max) value of f.

A local minimum of a function *f* is a value $f\,(c)$ that is less than or equal to all range values of f on some open interval containing c. If $f(c)$ is less than or equal to all range values of f then $f(c)$ is the **minimum** (or **absolute minimum**) value of f.

Local extrema are also called **relative extrema**.

Decide whether $f(x) = x^4 - 6x^2 + 8x$ local maxima or local minima.

(Use graphing utility to graph and locate.)

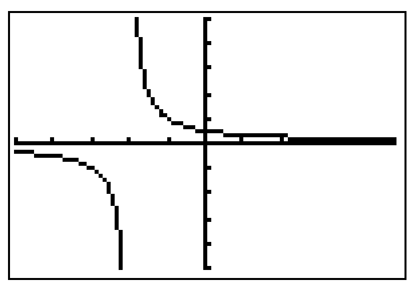


As Asymptotes – describe behavior of graph at its horizontal or vertical extremities:

$\lim\limits_{x \to a^+} f(x)$ means "x approaches a from the right"

$\lim\limits_{x \to a^-} f(x)$ means "x approaches a from the left"

Here we have $\lim\limits_{x \to -2^+} f(x) = \infty$ and $\lim\limits_{x \to -2^-} f(x) = -\infty$.



The line ***y* = *b*** is a **horizontal asymptote** of the graph of a function

$y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $+\infty$ or $-\infty$

In limit notation: $\lim\limits_{x \to -\infty} f(x) = b$ or $\lim\limits_{x \to +\infty} f(x) = b$.

The line $\boldsymbol{x = a}$ is a **vertical asymptote** of the graph of a function

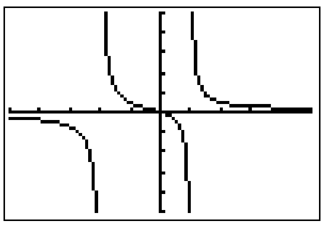
$y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ as x approaches a from either direction.

In limit notation: $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$.

Example:

Identify any horizontal or vertical asymptotes of the graph of $y = \dfrac{x}{x^2 + x - 2}$.

Graph:



$y = \dfrac{x}{x^2 + x - 2}$ can be written as $y = \dfrac{x}{(x+2)(x-1)}$. We know that the denominator of a fraction can't be zero, therefore $x \neq -2, 1$. Which is where there are vertical asymptotes, at x = -2 and x = 1.

If we look at the end behavior as *x* approaches $\pm\infty$ the *y* values of the function are getting closer to zero. Therefore there is a horizontal asymptote at *y* = 0.